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A HYBRID METHOD FOR THE NUMERICAL SOLUTION OF THE ELECTRON TRANSPORT EQUATION: THE REDUCED SOURCE METHOD

by

Gary S. Fraley, Kenneth Lee, and Michael A. Stroscio

ABSTRACT

The problem of correctly transporting the suprathermal electrons produced in laser-plasma interactions is complicated by the fact that the suprathermal mean free path varies over approximately ten orders of magnitude for typical laser fusion conditions. The reduced source method (RSM) offers a means of treating transport under such conditions. We derive the reduced source for the special case where the initial distribution is determined by multigroup diffusion. This special case represents an unnecessary restriction on the application of RSM to suprathermal electron transport.

I. INTRODUCTION

One of the major difficulties encountered in transporting suprathermal electrons produced by laser-plasma interaction[1-5] is due to the rapidly varying mean free path of the suprathermal electrons. The mean free path, λ_{mfp}, is related to the suprathermal velocity, v, and the background electron density, n_e, through $\lambda_{mfp} \sim v^4/n_e$. For typical laser-produced plasmas, λ_{mfp} varies over ten orders of magnitude from the underdense region outside of the critical surface to the overdense region inside of the critical surface. An adequate treatment of suprathermal electron transport must yield accurate solutions in the long mean free path (free streaming) limit as well as the short mean free path (diffusive) limit.

Previous attempts to treat suprathermal electron transport have utilized a wide variety of numerical schemes. (1) The multigroup diffusion method, where the distribution function is taken as a truncated angular expansion of Legendre polynomials, has been utilized in several treatments of SET.[6-9] These algorithms are suitable in short mean free path regions but are inadequate in the free streaming limit. (2) In addition, Monte Carlo treatments of SET are available.[6,10-14] The Monte Carlo treatment is, of course, adequate for all mean free paths; however, it is generally acknowledged that this method requires a relatively large number of numerical operations. (3) The method of discrete ordinates, where the transport equation is evaluated in a set of discrete angular directions, has been applied to SET and it is found to require fewer numerical operations than Monte Carlo transport.[14-19] This method is difficult to include in a Lagrangian hydrodynamics code. (4) Progress has been made in SET by utilizing a two-dimensional, two-fluid diffusion treatment that is certainly adequate in the diffusion regime.[10] (5) A hybrid model, which attempts to treat each velocity group in a given hydrodynamic cell by either diffusion or free streaming equations, has been developed.[20] This model does not provide transport solutions that are independent of the mean free path that is chosen to separate the long and short mean free path regimes. In addition, it is necessary to restrict the source distribution in ways that may not be consistent with the existence of plasma instabilities such as the Weibel instability.[21,22] (6) The transport of long mean free path electrons in the region of resonant fields has been accomplished by performing a "bounce-average."[23] Appropriate methods of interfacing this analysis with the transport in the short mean free path regime are unclear. (7) A relativistic transport equation[24] has been cast in the multigroup diffusion form.[25]

These methods of treating SET all have their own regime of validity. The most general method is Monte Carlo transport; however, there is a clear need to reduce the number of required computations. A method which is capable of (a) reducing the number numerical operations required and (b) yielding the accuracy of a full Monte Carlo scheme has been utilized by one of us (G.S.F.) in the area of radiation transport.[26,27] This method, the reduced source method (RSM), is discussed in the context of SET. Specifically, in Section II we present the transport equation appropriate to SET. In Section III we give a general discussion of the reduced source method and derive the reduced source for a special case. In Section IV we present a preliminary iteration scheme for the inclusion of electric fields.

II. SUPRATHERMAL ELECTRON TRANSPORT EQUATION

The equation describing the transport of suprathermal electrons has frequently been taken to be a Fokker-Planck approximation to the Boltzmann equation.[6,17,28] In this treatment, the Boltzmann equation for the distribution function $f(r,v,t,\mu)$ is taken as

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial r} + \frac{v(1-\mu^2)}{r} \frac{\partial f}{\partial \mu} + \frac{\vec{F}}{m_e} \cdot \frac{\partial f}{\partial \vec{v}} = \left(\frac{\delta f}{\delta t}\right)_{coll.} + S \tag{1}$$

for the case of spherical geometry in one dimension. In Eq. (1), $\mu = \cos\theta$, θ is the angle between the velocity and radius vectors, and r, v, and t are the radius, suprathermal velocity, and time, respectively. S is the source of suprathermal electrons and $\vec{F}$ is taken as $e\vec{E}$ to include the possiblity of electric field generation. The collision operator, $\left(\frac{\delta f}{\delta t}\right)_{coll}$, in the limit where collisions are dominated by small-angle scattering and the suprathermal-suprathermal interaction is assumed to be small, becomes the Fokker-Planck operator,

$$\left(\frac{\delta f}{\delta t}\right)_{coll.} = \frac{-4\pi e^4}{m_e^2} \ell n \Lambda \left\{ \frac{n_e}{v^2} \frac{\partial f}{\partial v} + \frac{n_e}{2} \frac{(Z+1)}{v^3} \frac{\partial}{\partial \mu} (1-\mu^2) \frac{\partial f}{\partial \mu} \right\}. \tag{2}$$

In Eq. (2), n_e is the cold electron density, Z is the charge of the cold ionic background, $\ell n \Lambda$ is a coulomb logarithm, and e and m_e are the electron charge and mass, respectively. We note that Ref. 17 contains a sign error in the leading term of the Fokker-Planck operator. Defining the flux of particles, $\phi = vf$, Eqs. (1) and (2) may be combined to give

$$\frac{1}{v}\frac{\partial \phi}{\partial t} + \mu \frac{\partial \phi}{\partial r} + \frac{(1-\mu^2)}{r}\frac{\partial \phi}{\partial \mu} - \frac{e|E|}{m_e}\left(\frac{\mu}{v}\frac{\partial \phi}{\partial v} - \frac{\mu\phi}{v^2}\right) = \left(\frac{\xi}{2}\right)\frac{1}{v^3}\frac{\partial \phi}{\partial v} - \left(\frac{\xi}{2}\right)\frac{\phi}{v^4} + \left(\frac{\xi}{2}\right)\frac{(Z+1)}{2v^4}\frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial \phi}{\partial \mu} + S, \tag{3}$$

where $\left(\frac{\xi}{2}\right) = -\frac{4\pi e^4 n_e}{m_e^2} \ell n \Lambda$. In our discussion of the RSM, it is convenient to cast Eq. (3) into the form

$$L(\phi) = S, \tag{4}$$

where

$$L(\phi) = \left(\frac{1}{v}\frac{\partial}{\partial t} + \mu\frac{\partial}{\partial r} + \frac{(1-\mu^2)}{r}\frac{\partial}{\partial \mu} - \frac{e|E|}{m_e}\left\{ \frac{\mu}{v}\frac{\partial}{\partial v} - \frac{\mu}{v^2} \right\} - \frac{\xi}{2}\frac{1}{v^3}\frac{\partial}{\partial v} + \frac{\xi}{2}\frac{1}{v^4} - \frac{\xi}{4}\frac{(Z+1)}{v^4}\frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial}{\partial \mu} \right) \phi \quad . \tag{5}$$

III. DERIVATION OF THE REDUCED SOURCE FOR SPECIAL CONDITIONS

The basic philosophy of the RSM has been discussed in detail in Refs. 26 and 27. It is assumed that an approximate solution to Eq. (4) has been obtained by any convenient means, e.g., multigroup diffusion. Let this approximate solution be ϕ_A. Then,

$$L(\phi) - L(\phi_A) = S - S_A = S_R, \tag{6}$$

where S_R is the reduced source and S_A is the approximate source corresponding to ϕ_A. From Eq. (6) the reduced source may be written as

$$S_R = S - L(\phi_A) \ . \tag{7}$$

Letting $\delta = \phi - \phi_A$, we obtain for a linear operator L,

$$L(\delta) = S_R \ . \tag{8}$$

(The operator L of Eq. (5) is not linear since the self-consistent electric field depends on the particle flux ϕ. Discussion of this point is deferred to Section IV.) From Eqs. (7) and (8) the basic idea of the RSM is apparent: an approximate solution ϕ_A is obtained by any convenient method and is utilized to calculate the reduced source, S_R, which is taken as the source in Eq. (8). Equation (8) is solved by the Monte Carlo method. For cases where ϕ_A is a good approximation to the exact solution, it follows that $S_R = S - S_A \ll S$; hence, the Monte Carlo solution of Eq. (8) requires many fewer numerical operations for an accurate solution than does Eq. (4). This last observation follows from the fact that the number of Monte Carlo simulation particles required for a solution of given accuracy is proportional to the square of the source.

Multigroup diffusion generally provides an accurate solution to ϕ in the short mean free path regime. In particular, the multigroup diffusion solution is expected to be accurate in the overdense region of the plasma, but is, of course, suspect in the plasma corona.

In the remainder of this report, we consider the special case where ϕ_A is approximated by the multigroup diffusion method. In this case it is possible that ϕ_A is a poor approximation to ϕ in the corona and we expect the Monte Carlo solution of Eq. (8) to require essentially the same number of numerical operations as a full Monte Carlo treatment. It is possible that an accurate solution for ϕ_A could be obtained in the corona by the techniques of Ref. 23. This possibility is being considered; however, this is not discussed here.

The multigroup diffusion technique in the P_1--approximation is applied by assuming a solution of the form

$$\phi_A = \phi_o + \phi_1\mu \quad , \tag{9}$$

where the expansion has been truncated after the P_1 term. Upon taking the first moment, $\frac{1}{4\pi}\int(\quad)d\Omega$, and second moment, $\frac{1}{4\pi}\int(\quad)\mu d\Omega$, of Eq. (4) with $\phi = \phi_A$, one obtains

$$\frac{1}{v}\frac{\partial\phi_o}{\partial t} + \frac{1}{3}\frac{\partial\phi_1}{\partial r} + \frac{2}{3}\frac{\phi_1}{r} - \frac{e|E|}{3mv}\left(\frac{\partial\phi_1}{\partial v} - \frac{\phi_1}{v}\right) - \frac{\xi}{2}\frac{1}{v^3}\frac{\partial\phi_o}{\partial v} + \frac{\xi}{2}\frac{1}{v^4}\phi_o = \langle S\rangle, \tag{10}$$

$$\frac{1}{v}\frac{\partial\phi_1}{\partial t} + \frac{\partial\phi_o}{\partial r} - \frac{e|E|}{mv}\left(\frac{\partial\phi_o}{\partial v} - \frac{\phi_o}{v}\right) - \frac{\xi}{2}\frac{1}{v^3}\frac{\partial\phi_1}{\partial v} + \left(\frac{Z+2}{2}\right)\frac{\xi}{v^4}\phi_1 = 3\langle\mu S\rangle , \tag{11}$$

where $\langle S\rangle = \frac{1}{4\pi}\int S d\Omega$, $\langle\mu S\rangle = \frac{1}{4\pi}\int\mu S d\Omega$, and we have used

$$\frac{1}{4\pi}\int d\Omega = 1, \ \frac{1}{4\pi}\int\mu^2 d\Omega = \frac{1}{3}, \ \frac{1}{4\pi}\int\mu^3 d\Omega = 0, \ \frac{1}{4\pi}\int\mu d\Omega = 0, \text{ and } \frac{\partial\phi}{\partial\mu} = \phi_1 \tag{12a-e}$$

in obtaining the moments. Equations (10) and (11) represent, respectively, the first and second moments of the transport equation. These equations are rewritten as

$$\tau = \langle S\rangle - \frac{1}{v}\frac{\partial\phi_o}{\partial t} - \frac{1}{3}\frac{\partial\phi_1}{\partial r} - \frac{2}{3}\frac{\phi_1}{r} + \frac{e|E|}{3mv}\left(\frac{\partial\phi_1}{\partial v} - \frac{\phi_1}{v}\right) \quad , \tag{13}$$

$$\rho = 3 \langle \mu S \rangle + \frac{e|E|}{mv}\left(\frac{\partial \phi_o}{\partial v} - \frac{\phi_o}{v} \right) , \tag{14}$$

where τ (ρ) is defined by comparison with Eq. (10) [Eq.(11)]. From Eqs. (4) and (7) with $\phi = \phi_A$, one obtains

$$S_R = S - \left[\frac{1}{v}\frac{\partial \phi_o}{\partial t} + \mu^2 \frac{\partial \phi_1}{\partial r} + \frac{(1-\mu^2)}{r}\phi_1 - \frac{e|E|\mu}{mv}\left(\frac{\partial \phi_o}{\partial v} - \frac{\phi_o}{v}\right) - \frac{e|E|}{mv}\mu^2\left(\frac{\partial \phi_1}{\partial v} - \frac{\phi_1}{v}\right) + \tau + \rho\mu \right] = [S - \langle S \rangle - 3\mu \langle \mu S \rangle] + (\mu^2 - \mu^2_o)\left[\frac{\phi_1}{r} - \frac{\partial \phi_1}{\partial r}\right]$$
$$+ (\mu^2 - \mu^2_o)\frac{e|E|}{mv}\left(\frac{\partial \phi_1}{\partial v} - \frac{\phi_1}{v}\right) , \tag{15}$$

where $\mu^2_o = 1/3$. This reduced source for the electron transport equation differs from that derived for radiation transport[26,27] in three respects: (1) an anisotropic source must be included as is represented by the presence of the first three terms in Eq. (15); (2) the electric field must be included in any self-consistent one-dimensional formulation as is manifest in Eq. (15) by the presence of the terms containing $|E|$; and (3) the remaining terms in Eq. (15), other than electric field or source terms, differ from the corresponding radiation transport reduced source as a result of the basic differences in the electron and radiation transport equations. The electric fields in Eq. (15) have been treated purely formally to this point. That is, the electric field has been taken as given independently of the particle flux ϕ. This is, of course, not the case and this must be corrected by iteration of the electric field as a function of the particle flux.

IV. A PRELIMINARY ITERATION SCHEME FOR THE INCLUSION OF ELECTRIC FIELDS

The electric field depends on the current, J_H, associated with the suprathermal electron flux ϕ and in turn on the cold return current, J_C. The first step in the proposed iteration scheme is to utilize the multigroup diffusion approximation to ϕ, namely ϕ_A, in order to calculate $|E^0| = |E(\phi_A)|$. This value of $|E|$ is then inserted into L and Eq. (8) is solved by the Monte Carlo method. The new value of ϕ, $\phi^{(1)}$ is then used to calculate $|E^1| = |E(\phi^{(1)})|$. This iteration is repeated until

$$\left| \, |E^n| - |E^{n-1}| \, \right| < \varepsilon_1 \quad , \tag{16a}$$

$$\left| \phi^n - \phi^{n-1} \right| < \varepsilon_2 \quad , \tag{16b}$$

where n refers to the index defining the iteration and ε_1 and ε_2 represent user-defined convergence criteria.

V. CONCLUSION AND SUMMARY

The RSM provides a consistent method of treating SET in both the short and long mean free path regimes of a laser-produced plasma. In addition to reducing the time required by a full (but accurate) Monte Carlo simulation, the RSM provides a method of interfacing different methods of solution. We have restricted our discussion to the case where an approximate solution is obtained by the multigroup diffusion approximation. In this case the transport solution in the plasma corona is suspect and we expect to do essentially a full Monte Carlo treatment in the corona.

The RSM is by no means limited to this treatment. For example, a promising approach which is being investigated is based on using a multigroup diffusion solution for ϕ_A in the short mean free path regime and a "bounce-averaged" solution for ϕ_A in the long mean free path regime where resonant fields and the reflecting plasma sheath play major roles in suprathermal electron transport.

In this report, we have presented a general discussion of the RSM for SET and have given an example that obtains when ϕ_A is determined by multigroup diffusion. This is not necessarily the best scheme available. We are presently investigating other techniques of applying the RSM to SET.

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